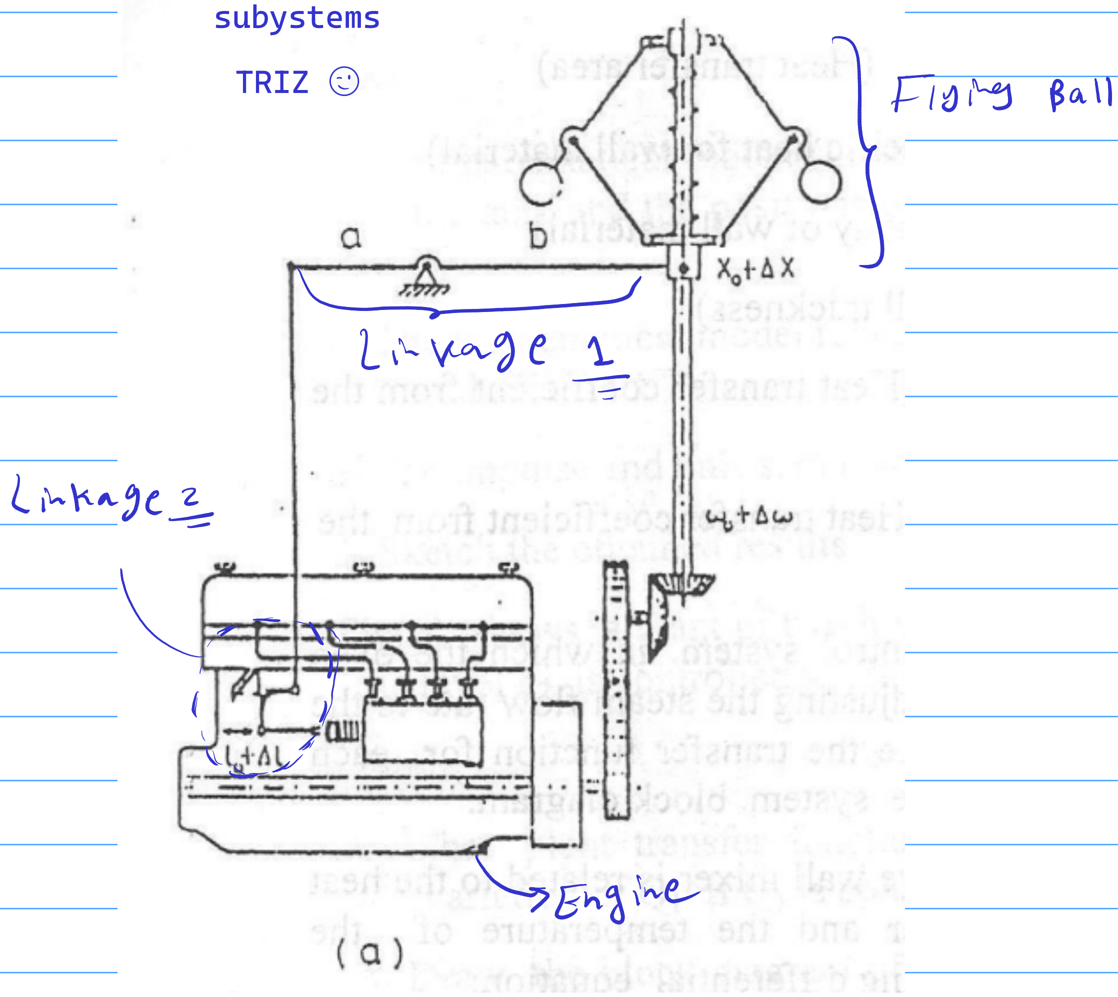
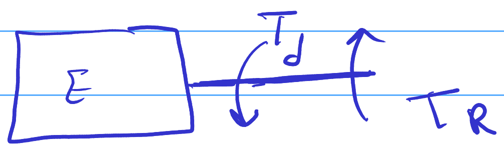
Derive the mathematical models of each component in the systems shown in fig (1) then construct the equivalent closed loop system on each case.

Segment the system into subystems

TRIZ ☺



* Engine



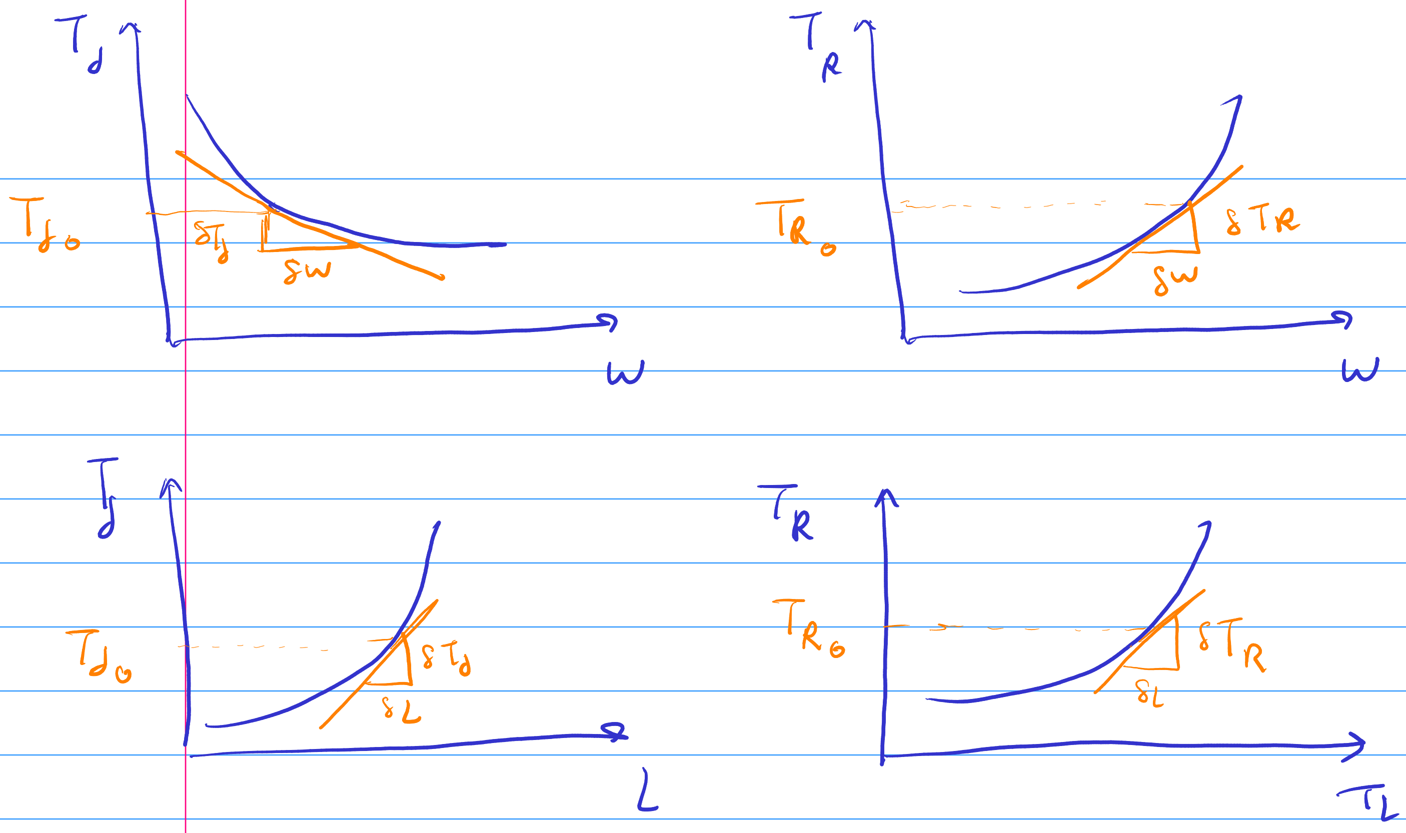
$$T_{net} = J_{eq}\,\dot{\omega} = T_d - T_R$$

$$T_d = f(\omega, l)$$

Developed torque is a function of angular speed and throttled rack length

$$T_R = g(\omega, T_L)$$

Resistive toruqe is a function of angular speed and load torque



f and g are non-linear functions

use taylor expansion
for lienarization

$$T_d \simeq T_{d_0} + \frac{\partial T_d}{\partial \omega} \partial \omega + \frac{\partial T_d}{\partial L} \partial L$$

$$T_R \simeq T_{R_0} + \frac{\partial T_R}{\partial \omega} \partial \omega + \frac{\partial T_R}{\partial T_L} \partial T_L$$

you can read the above two equations as follow:

- developed torque is equal to deveoped torque at equilibrium + tiny changes in developed torque caused by tiny changes in angular speed and tiny changes in throttle rack length

- resistive torque is equal to resistive torque at equilibrium + tiny changes in resistive torque caused by tiny changes in angular speed

since we are linearizing Td and TR, then the derivatives are const:

$$\frac{\partial T_d}{\partial L} = c_1 \qquad \frac{\partial T_d}{\partial \omega} = C_2 \qquad \frac{\partial T_R}{\partial T_L} = c_3 \qquad \frac{\partial T_R}{\partial \omega} = c_4$$

$$T_d = T_{d_0} + c_1 \Delta L + c_2 \Delta w$$

$$T_R = T_{R_0} + c_3 \Delta T_L + c_4 \Delta w$$

at equilibrium: $T_d = T_R \longrightarrow T_{d_0} = T_{R_0}$

$$T_{net} = J_{eq}\dot{w} = T_d - T_R = c_1 \Delta L + (c_2 - c_4)\Delta w - c_3 \Delta T_L$$

$$\xrightarrow{\mathcal{L}} T(s) = J_{eq}\, s w = c_1 L(s) + [c_2 - c_4]\, w(s) - c_3 T_L(s)$$

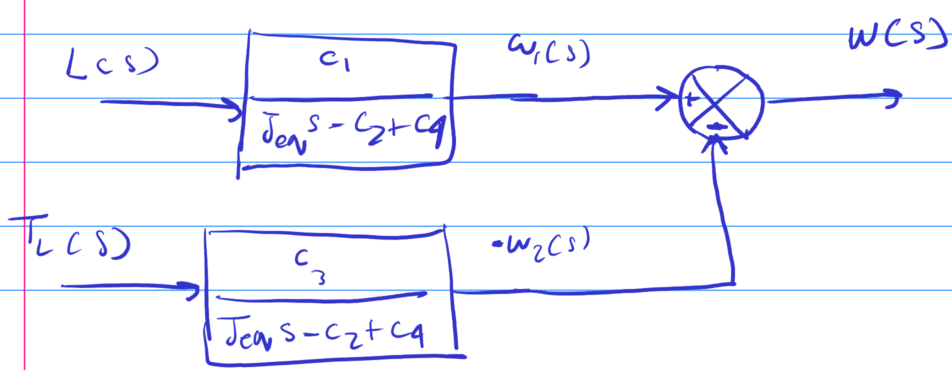
$$[J_{eq}\, s - c_2 + c_4]\, w(s) = c_1 L(s) - c_3 T_L(s)$$

to get a transfer function for w(s), we need to use superposition princple

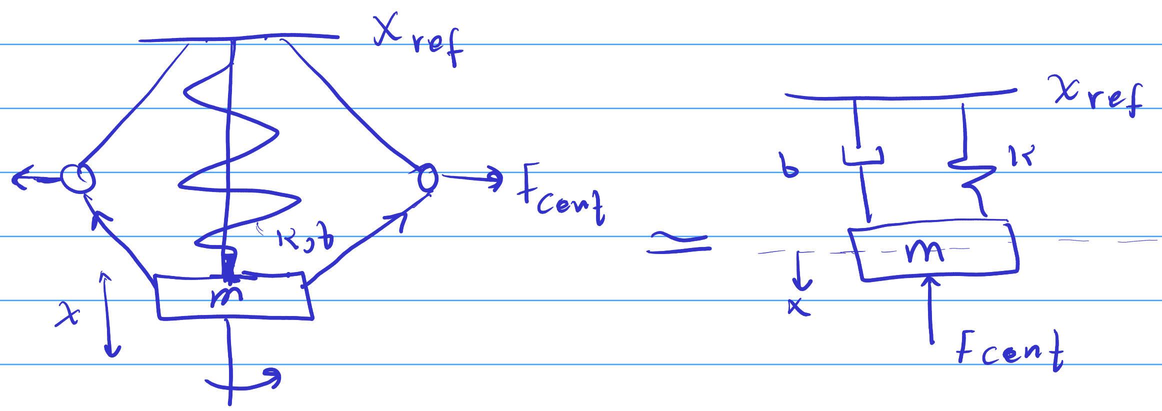
@ $T_L(s) = 0 \longrightarrow \dfrac{w_1(s)}{L(s)} = \dfrac{c_1}{J_{eq}\, s - c_2 + c_4}$

@ $L(s) = 0 \longrightarrow \dfrac{w_2(s)}{T_2(s)} = \dfrac{-c_3}{J_{eq}\, s - c_2 + c_4}$

to get w(s), we need to sum w1(s) (the influence of L(s) on w(s)) and w2(s) (the influence of Tl(s) on w(s))



* Flying Ball



Note:
$\dot{x}_{ref} = 0$
$\ddot{x}_{ref} = 0$

$$F_{net} = m\ddot{x} = F_{cent} - k(x - x_{ref}) - b\dot{x}$$

Fcent, centripital force, is a function of w

let:

$$F_{cent} \propto w$$

$$F_{cent} = C_{cent}\, w$$

$$m\ddot{x} = C_{cent}\, w - k(x - x_{ref}) - b\dot{x}$$

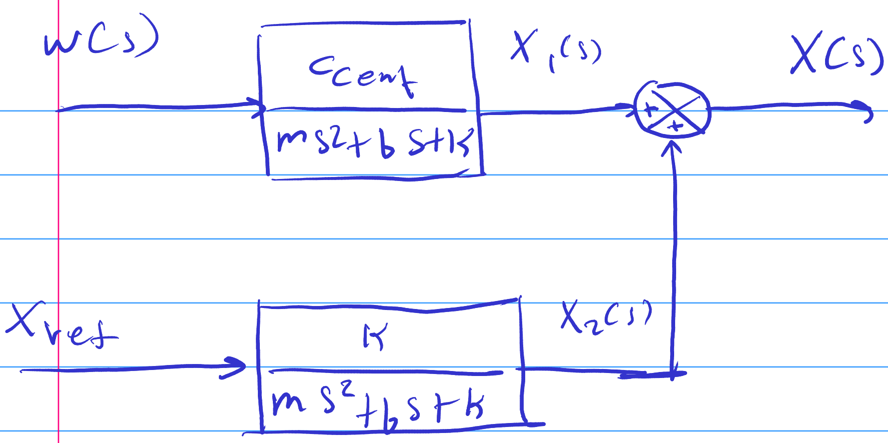
$$\mathcal{L} \rightarrow \left[ms^2 + bs + k\right] X(s) = k X_{ref}(s) + C_{cent} W(s)$$

to get a transfer function for x(s), we need to use superposition

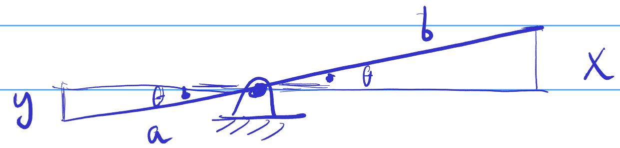
@ $X_{ref}(s) = 0 \longrightarrow \dfrac{X_1(s)}{W(s)} = \dfrac{C_{cent}}{ms^2 + bs + k}$

@ $W(s) = 0 \longrightarrow \dfrac{X_2(s)}{X_{ref}(s)} = \dfrac{k}{ms^2 + bs + k}$

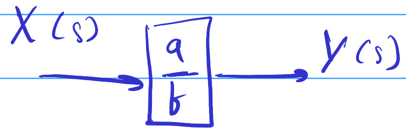
to get X(s), we need to sum X1(s) (the influence of w(s) on X(s)) and X2(s) (the influence of Xref(s) on X(s))



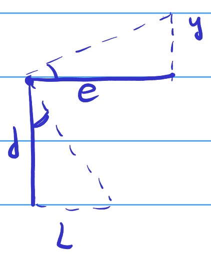
* Linkage $\underline{1}$



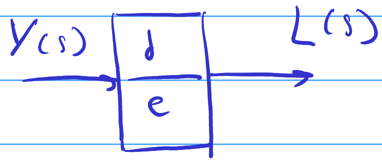
$$\sin\theta = \frac{y}{a} = \frac{x}{b} \xrightarrow{\mathcal{L}} Y(s) = \frac{a}{b} X(s)$$



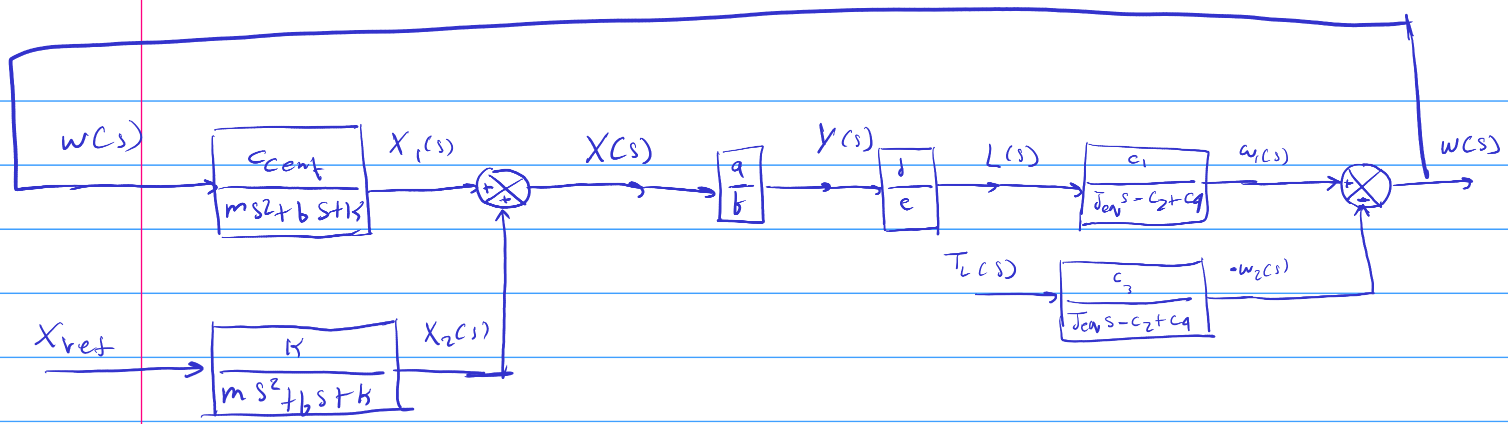
* Linkage $\underline{2}$



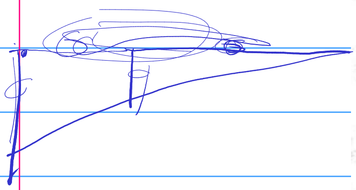
$$\tan\theta = \frac{y}{e} = \frac{L}{d} \xrightarrow{\mathcal{L}} L(s) = \frac{d}{e} Y(s)$$

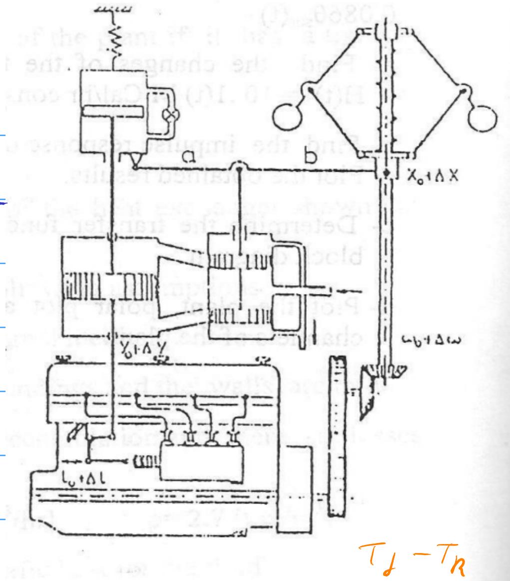


system



w(s)
$\frac{c_{cemf}}{ms^2+bs+k}$
$X_1(s)$
X(s)
$\frac{a}{b}$
y(s)
$\frac{d}{e}$
L(s)
$\frac{c_1}{J_{eq}s-c_2+c_4}$
$\omega_1(s)$
w(s)
X_{ref}
$\frac{k}{ms^2+bs+k}$
$X_2(s)$
$T_L(s)$
$\frac{c_3}{J_{eq}s-c_2+c_4}$
$\omega_2(s)$





(d)

$T_d - T_R$

$J\dot{w} = T_{net}$ $\quad T_d = f(L, w)$ $\quad T_R = g(T_L, w)$

$c_1 \quad c_2 \quad c_3 \quad c_4$

* Engine: $J s w(s) = c_1 L(s) + c_2 w(s) - c_3 T_L(s) - c_4 w(s)$

$$w(s) = \frac{c_1}{Js + c_4 - c_2} L(s) - \frac{c_3}{Js + c_4 - c_2}$$

$m, b, k \leftarrow f(x) = \overset{C_{cent}}{F(w)} - F(x_{ref}) \rightarrow k$

* Flyball: $[ms^2 + bs + K] X(s) = C_{cent} W(s) - K X_{ref}(s)$

$$X(s) = \frac{C_{cent}}{ms^2+bs+K} W(s) - \frac{K}{ms^2+bs+K} X_{ref}(s)$$

$+X \frac{a}{b+a} \quad -Z \frac{b}{b+a}$

* Linkage: $Y(s) = \frac{a}{b+a} X(s) - \frac{b}{b+a} Z(s)$

$b \leftarrow f(R) \qquad f(z) \rightarrow m, b, k$

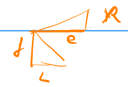
* Spool valve: $b_2 s R(s) = [ms^2 + b_2 s + k_2] Z(s)$

$$R(s) = \frac{ms^2 + b_2 s + k_2}{b_2 s} Z(s)$$

$c_h \qquad h(y) \rightarrow c_5$

* Tank: $A s R(s) = c_5 Y(s)$

$$R(s) = \frac{c_5}{As} Y(s)$$



* Linkage2: $R(s) = \frac{e}{d} L(s)$

Variables:

$w(s), X(s), Y(s), Z(s), R(s), L(s)$

Engine:

$$w(s) = \frac{C_2}{Js+C_5} L(s) - \frac{C_3}{Js+C_5} T_{ext}(s)$$

flyball Sensor:

$$X(s) = \frac{K}{ms^2+bs+K} X_{Ref}(s) + \frac{C_{cent}}{ms^2+bs+K} w(s)$$

Lever:

$$Y(s) = \frac{a}{a+b} X(s) - \frac{b}{a+b} Z(s)$$

Spool valve:

$$q_{in} = \cancel{q_o} + q_{stored} \Rightarrow q_{in} = A\frac{dR}{dt} = C_y * y$$

$$AsR(s) = C_y Y(s)$$

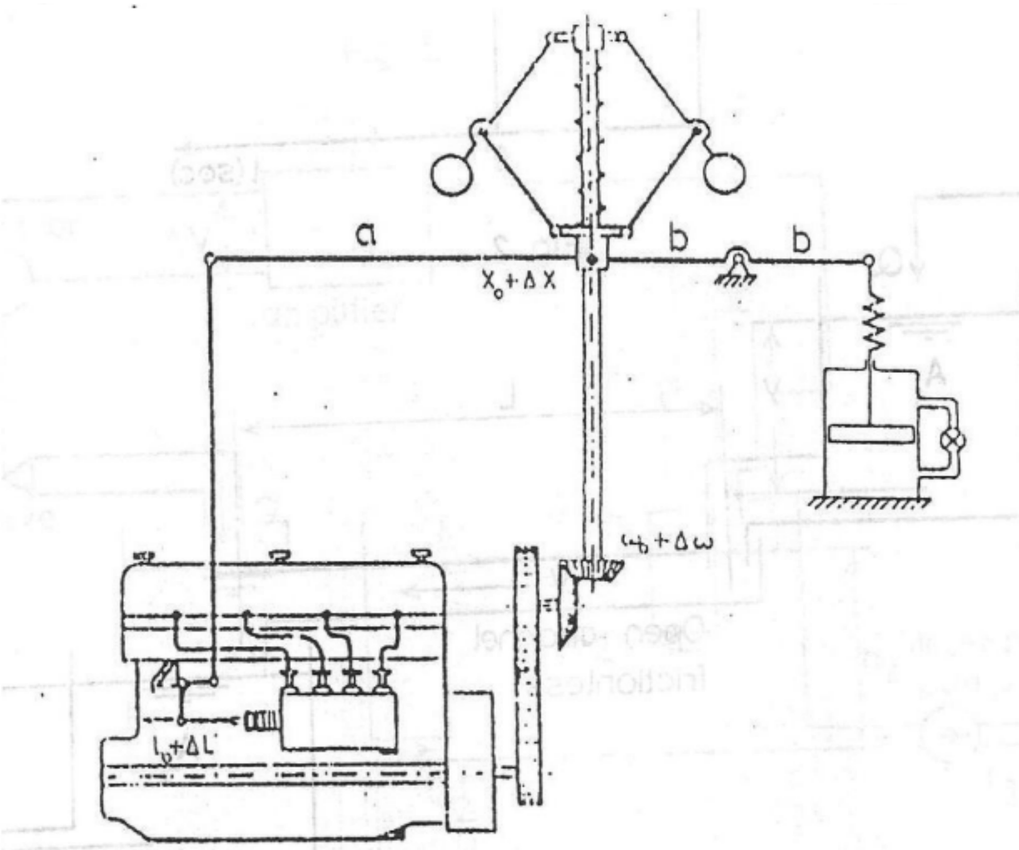
$$R(s) = \frac{C_y}{As} Y(s)$$

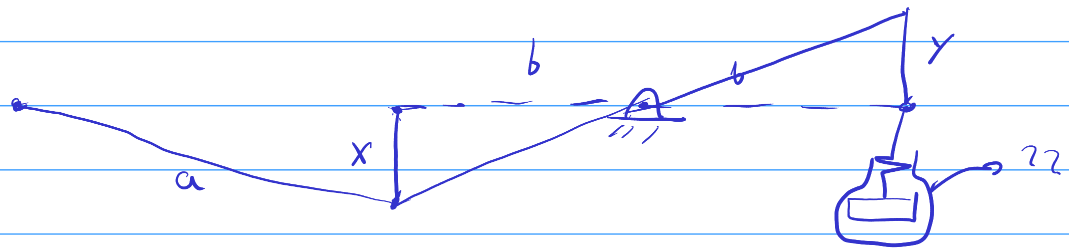
&

$$Z(s) = \frac{b_2 s}{m_{assy} s^2 + b_2 s + k_2} R(s)$$

&

$$L(s) = \frac{d}{c} R(s)$$


a
b
b
$X_o + \Delta X$
A
$\omega_o + \Delta\omega$
$L_o + \Delta L$


b
b
Y
X
a
??